

Optimization of Land Grading Technique by A Mathematical Modeling

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Abstract

An optimization mathematical model of land grading technique based on the theorem of least square is developed. The methodology is fundamentally depended upon the minimization of the summation of cuts and fills. Ancient Babylon City of (17,562,500m²) in area is undertaken as a modeled area. Leveling readings as a field work of random remarks are performed within the area. The model domain is discretized into (no. of columns) NC = 20 and (no. of rows) NR = 21 with grid spacing of (250 m) in both x, and y directions. The data is entered into the model as input data. The resulting optimum plane is obtained with a summation of cuts and fills of (39.27m³ fill).

1 Introduction

Land grading is reshaping of the existing land surface in accordance with a plan as determined by engineering survey and layout. The purposes of land grading technique are to protect land surface against erosion, sedimentation, infiltration, direct surface runoff and allowance for simpler landscaping and/or construction and large culture projects. *The optimum land grading technique is the one of minimum amount of cut and fill to minimize labor, time, and cost provided that the purpose of that slope are fulfilled.*

Safa and Ahmed (1990) present an optimization technique to perform land-grading designs by selecting a best-fit curve or plane surface. The profiles of such surfaces along either of the two major directions are assumed to be represented by a general power function. The main goal of this technique is to minimize the volumes of earth work required while obtaining a desirable smooth surface. Furthermore, the fitted surface can be subjected to a series of constraints: Limiting the slope at any point; choosing the desired surface shape, i.e., concave, convex, or plane; and limiting the elevation of the graded surface to allow gravity irrigation from a water sources. Nonlinear programming has been used to perform the fitting procedure.

Kenneth (1988) outlined that Level basin is one of the factors bearing on irrigation system selection. Level basins simplify water management, since the irrigator need only supply a specified volume of water to the field. With adequate stream size, the water will spread quickly over the field, minimizing non-uniformities in inundation time. Basin irrigation is most effective on uniform soils, precisely leveled

Salassi (2000) Presents cost estimates of precision grading sugarcane fields for which the sugarcane producer purchases the laser-leveling and dirt-moving equipment and performs the work with farm labor. Both variable and fixed costs associated with precision grading are estimated on a per hour of operation basis as well as costs per acre and per cubic yard of dirt moved. He estimated the costs of precision land grading of Sugarcane Fields to increase production. He estimated the cost of equipments per hour and then this used to estimate the cost per acre. It is found that the cost of precision land grading \$153.72 per acre.

Joe et al (1998) A study was undertaken to compare yield variations in fields to topographic feature, specifically low spots that cause surface ponding of water. The interest in topography within the field of

precision agriculture as a research project stems from the relative ease and low cost of data collection. Professional services charge from \$3 - \$5/acre and provide a high density of data (4,000 points/acre).

Liu and Zhang (2002) argue the urban land grading system based on the self-developed GIS software. The system establishes the relevant data structure and the empirical formulas of the affecting factor and economic data of land use. Besides, the system can calculate the service radius, sum and land grade automatically. Furthermore, it can do statistics and query on various land information and show the result of land grading with spatial and attribute data. This paper illustrates the result by giving an example of the commercial land grading of the urban area of Wuhan City by the urban land grading system.

Zhao et al (2009) Aiming at the insufficiencies of traditional agricultural land grading methods, this study discussed the process and technical route of agricultural land grading based on decision tree analysis method and GIS, constructed an agricultural land grading model based on MATLAB and decision tree C4.5 algorithm. Furthermore, we took Luanwan village of Pingyin county in China for the empirical study, selected seven indicators as the test attributes, predicted agricultural land grade on support of this model, and expressed the rules in the quantitative way. The results showed that agricultural land grading model based on decision tree which is coded in M-language of MATLAB doesn't rely on the empirical knowledge. It has the ability of self-learning, and the gained rules are easy to be understood. Moreover, the high rate of accuracy will be able to meet the requirements of evaluation.

All land owner and developers considered strictly that land grading cost is a major part of the project total cost and future revenues. In this research, the minimization of land grading cost may be formulated by the following objective function:

$$\text{Min } J = (\sum \text{fills} + \sum \text{cuts}) \dots\dots\dots(1)$$

The difference between the present work and [1] is that the later used an optimization technique to fit the plane surface whereas in this research a mathematical solution based on the least square method is used to get the requested surface with minimum cut and/or fill requirements.

2 Purpose of the Study

A combination process of a numerical background and computer programming is adapted to develop a land grading technical model is applicable for regular and irregular surfaces.

3 Case Study

Frequently, in wide and rugged areas where a flattened and graded land with a specified plane is requested, the designer faces constantly the problem of choosing the best plane to produce minimum cut and fill earth work. Consequently, the current study serves this target

4 Preparation of A Mathematical Model

4.1 MATHEMATICAL BACKGROUND

Ground surface mathematically represents a 3D function i.e $H = f(x, y)$, where H is the altitude of land in x and y dimension. In simple case a straight plane may be fitted by eye, but if the points are scattered, this becomes unreliable and it is better to employ mathematical principles. Erwin (1962) a widely used procedure of such types of problems is the *least square method*.

The function of the fitted plane may be assumed as $Z = f(x, y, s_x, s_y)$; where z is altitude of the fitted plane and s_x, s_y are the slopes of the fitted plane in x and y direction respectively, then cut /or fill = $H - Z = f(x, y, s_x, s_y)$.

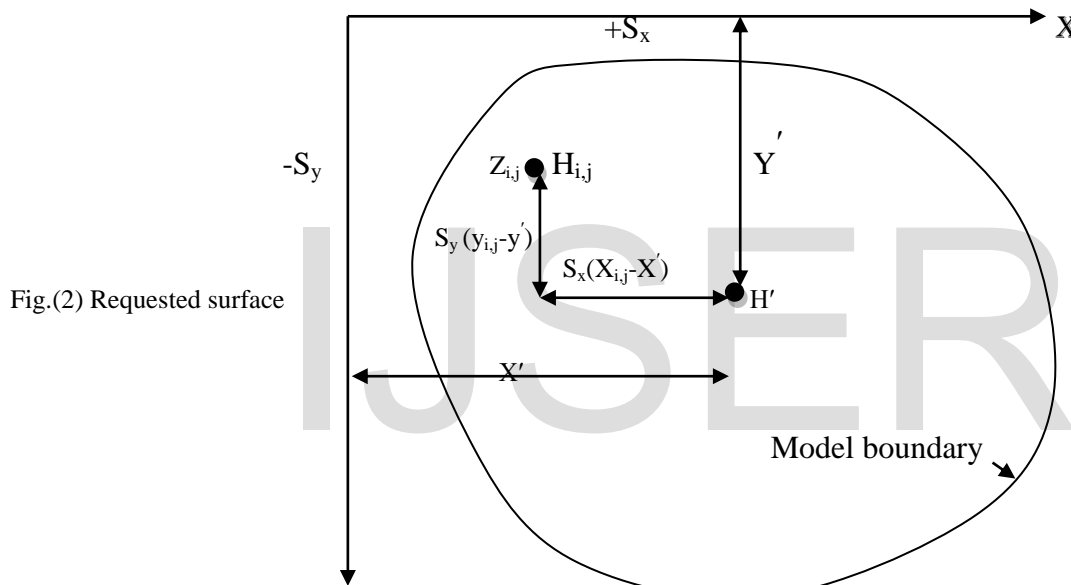
In a matrices system; cut and/or fill = $H_{ij} - Z_{ij}$, and if a function J is assumed to be a summation of fill and cut squares

$J = \sum \text{fill}^2 + \sum \text{cut}^2$ (the cuts and fills are lifted to the power two to avoids the problem of minus sign and termination of the term J)

Clearly, J depends on s_x and s_y . Minimization of J by simple differentiation with respect to both s_x and s_y slopes in eqs.(2)

$$\frac{\partial J}{\partial s_x} = 0, \quad \frac{\partial J}{\partial s_y} = 0 \quad \dots \dots \dots (2)$$

4.2 CONSTRUCTION OF THE REQUESTED SURFACE



Referring to Fig.(2) The new elevation $Z_{i,j}$ of arbitrary node within the model domain is:

$z_{i,j} = H' + S_x(X_{i,j} - X') + S_y(y_{i,j} - y')$, whereas the value of cut/or fill is
 $C/orF = H' + S_x(X_{i,j} - X') + S_y(y_{i,j} - y') - H_{i,j}$ where $H_{i,j}$ is the natural ground level.

$$J = \sum_{i=1}^{NR} \sum_{j=1}^{NC} (H' + S_x(X_{i,j} - X') + S_y(y_{i,j} - y') - H_{i,j})^2 \dots \dots \dots (a)$$

By carrying out the derivations of Eq.(a) with respect to both S_x & S_y , the following two simultaneous equations are obtained:

$$\frac{\partial J}{\partial s_x} = 2 \sum \sum (H' + S_x(X_{i,j} - X') + S_y(y_{i,j} - y') - H_{i,j})(X_{i,j} - X') \dots \dots \dots (a1)$$

$$\frac{\partial J}{\partial S_y} = 2 \sum \sum (H' + S_x(X_{i,j} - X') + S_y(y_{i,j} - y') - H_{i,j})(y_{i,j} - y') \dots \dots \dots (a2)$$

By rearranging, generation the summation and equating to zero:

$$[\sum \sum H' - \sum \sum H_{i,j} + S_x(\sum \sum X_{i,j} - \sum \sum X') + S_y(\sum \sum y_{i,j} - \sum \sum y')](\sum \sum X_{i,j} - \sum \sum X') = 0 \dots (a3)$$

$$[\sum \sum H' - \sum \sum H_{i,j} + S_x(\sum \sum X_{i,j} - \sum \sum X') + S_y(\sum \sum y_{i,j} - \sum \sum y')](\sum \sum y_{i,j} - \sum \sum y') = 0 \dots (a4)$$

$(\sum \sum X_{i,j} - \sum \sum X') = 0$ equals $(\sum \sum X_{i,j} - NX') = 0$ which leads to $X' = \frac{\sum \sum X_{i,j}}{N}$ and

$(\sum \sum y_{i,j} - \sum \sum y') = 0$ equals $(\sum \sum y_{i,j} - Ny') = 0$ which leads to $y' = \frac{\sum \sum y_{i,j}}{N}$

Set $S_y(\sum \sum y_{i,j} - \sum \sum y') = 0$, Eq.(a3) may be reduced to:

$$[\sum \sum H' - \sum \sum H_{i,j} + S_x(\sum \sum X_{i,j} - \sum \sum X')] = 0. \text{ Multiplying by } X_{i,j} \text{ to obtain}$$

$$S_x(\sum \sum X_{i,j}^2 - NX'^2) = \sum \sum (HX)_{i,j} - NX'H'$$

Solve the final form for S_x to obtain Eq.(3) and Eq.(4).

$$S_x = \frac{\sum_{i=1}^{NR} \sum_{j=1}^{NC} (HX)_{ij} - NX'H'}{\sum_{i=1}^{NR} \sum_{j=1}^{NC} X_{ij}^2 - NX'^2} \dots \dots \dots (3)$$

$$S_y = \frac{\sum_{i=1}^{NR} \sum_{j=1}^{NC} (Hy)_{ij} - Ny'H'}{\sum_{i=1}^{NR} \sum_{j=1}^{NC} y_{ij}^2 - Ny'^2} \dots \dots \dots (4)$$

Where: x' and y' are the centroid of the domain.

H' : is the average of natural altitude of the ground.

N : is the number of selected remarks.

Equations (3) and (4) may be used to estimate the optimum slopes of the graded surface.

4.3 IMPLEMENTATION OF GRID SYSTEM DESIGN

The first step in modeling job is to discretize the system by superimposing a mesh of finite grid over the map of the area. The size of mesh or in other words the number of rows and columns that is to be adapted depends on the required accuracy. The total dimensions of the grids are defined by NC (the number of columns), and NR (the number of rows) of the model. Non-uniform grid spacing may be used; however uniform spacing is used in the present work because the area is relatively large. The chosen number of columns and rows in the present model is NC = 3 and NR = 4. Uniform grid spacing of (20 m) is selected in X and Y directions. Fig.(1) shows the discretization system of a domain, slopes direction, and calculation procedure.

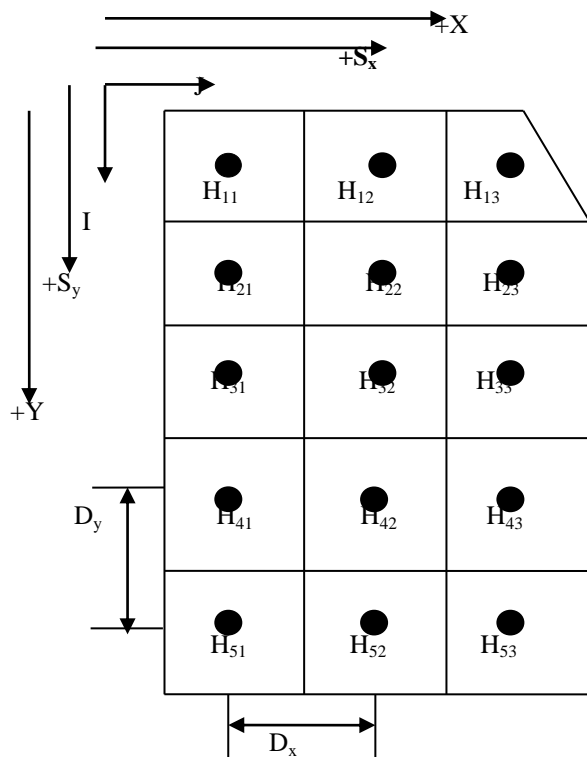


Fig.(1) Descretization System, Slopes Direction, and Calculation Procedure

4.4 COMPUTER PROGRAMMING AND ITS VALIDITY

A program has been written for Land simulation by a modeling process. It is written in a Fortran Language for its high executive capacity. It is written in easier manner to be flexible and easy to be modified. The derivation of the basic equations; (3) and (4) is based on a simple partial derivation. A convergence test for errors of the model is proved and verified. The simplification is mainly made in the input/output data settings and filling, rather than in the computational technique which is rather a straightforward procedure. *The error term is defined as the maximum sum of the cuts and fills for all setting node values during a calculation process.*

An initial level values for the measured grid is set out in a level data file; whereas the subsequent estimated requested levels and the amount of cut and fill for each individual grid are inserted in the output data files. The basic land grading program is listed in Appendix-A. A general framework of the model is shown in the flowchart of Fig.(3).

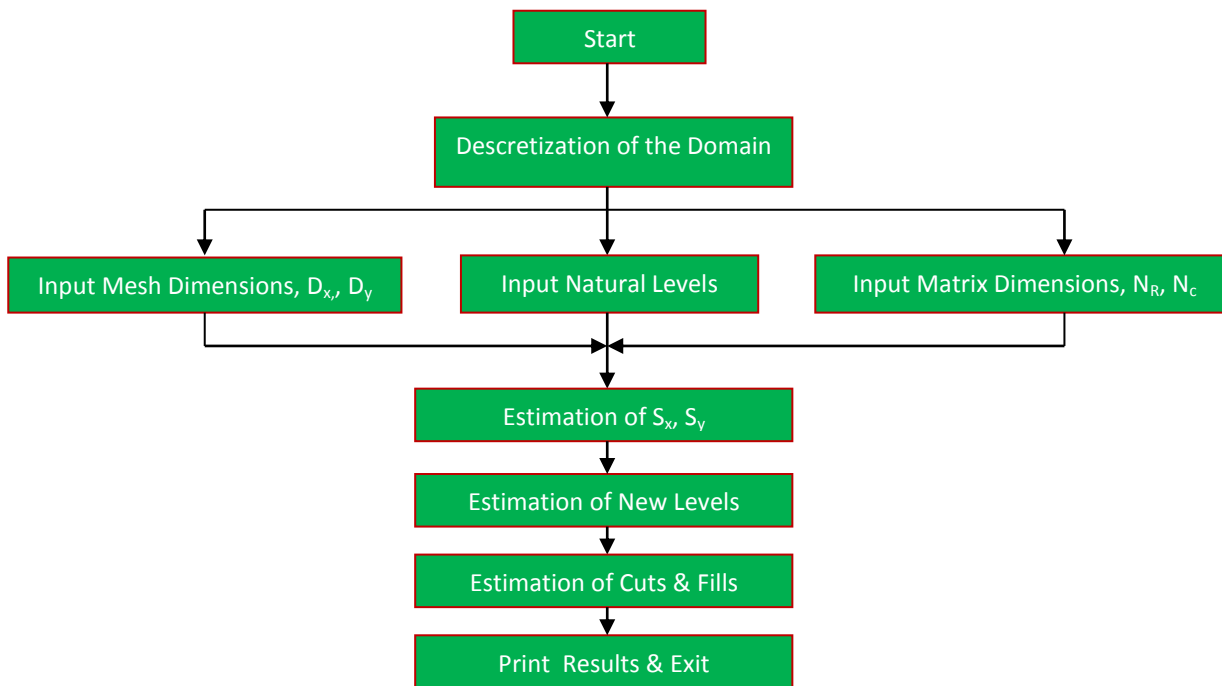


Fig.(3) Flowchart showing a General Model Framework

5 Simulated Area and Its Geographic Description

The area of Ancient Babylon City is selected to be a sample of the study. The area is characterized with a rugged and undulation nature. It is located between Longitudes ($44^{\circ} 24' 45''$ - $44^{\circ} 26' 15''$) and latitudes ($32^{\circ} 31' 30''$ - $32^{\circ} 32' 51''$). It is bounded by Al Hillah River from the West, whereas An Artificial ditch is located to the east. Fig.(4) represents a location map of the study area. The area undertaken in this mathematical simulation is about $17,562,500\text{m}^2$ in size. The maximum and minimum ground levels are about 60m and 32m above mean sea levels respectively.

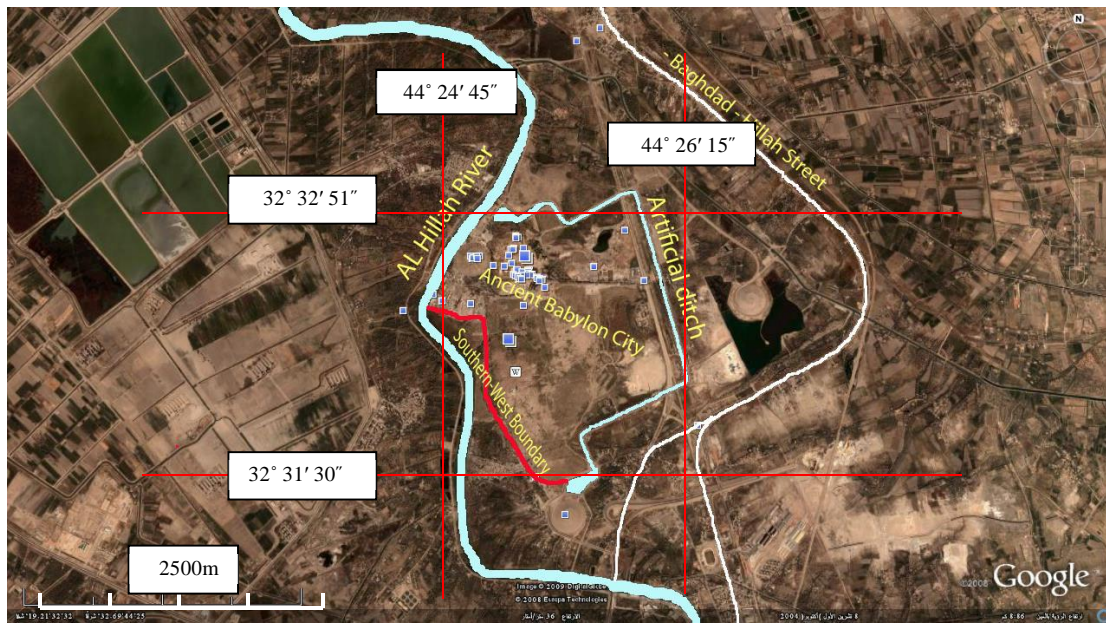


Fig.(4) Location Map of Ancient Babylon City (Areal View, Based on Google Earth 2008)

Topography

A land leveling at the considered area is carried by the author, using a *Google Earth Technology*, *leveling instrument* and *GPS Device* to obtain natural ground levels, and longitude and latitude coordinates. The topographic features of the area are represented graphically in Fig.(5). 3D sketch of Fig. (6)Presents the contour lines of natural topography of the interested area.

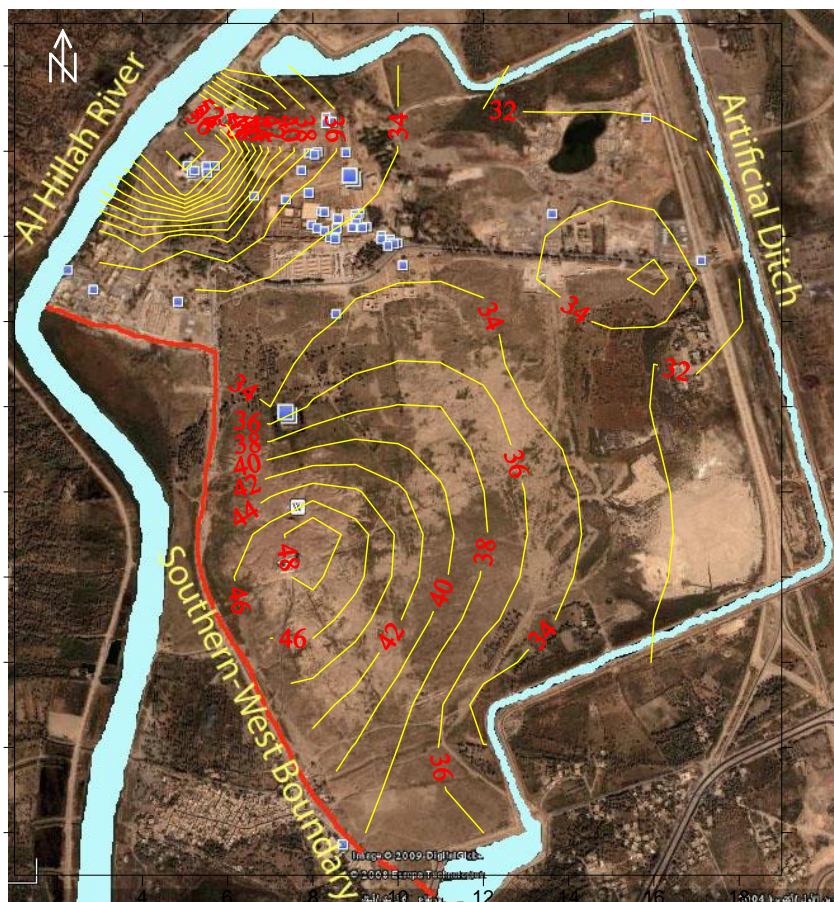


Fig.(5) Topographic Contourmap of Ancient Babylon City

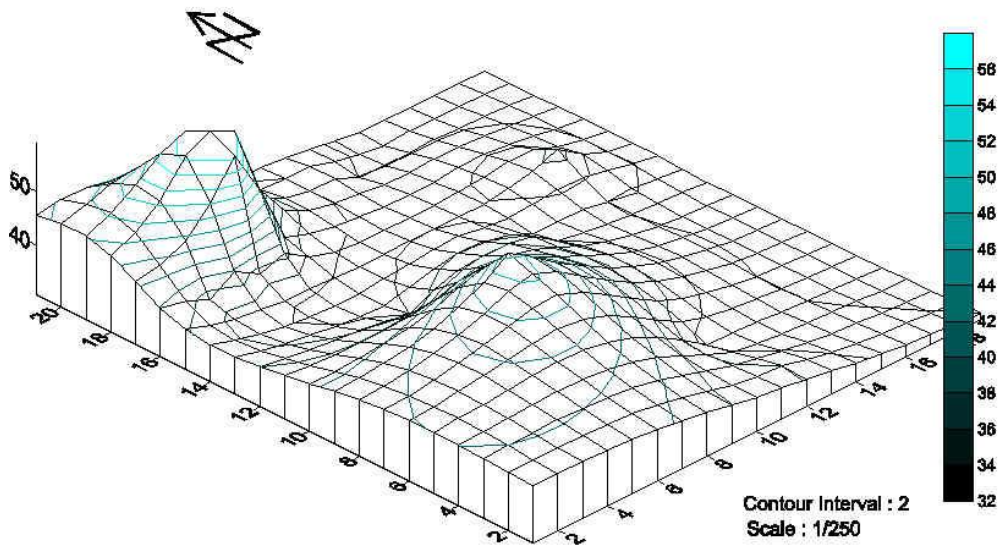


Fig.(6) 3D Contour Map of Ancient Babylon City

5.2 Mesh Design of the Domain

A square paper is superimposed over the geographic map to discretize the domain of the considered area into a suitable number of meshes. It is found that the number of columns $NC = 20$ and the number of rows $NR = 21$. The current discretization is shown in Fig.(7).

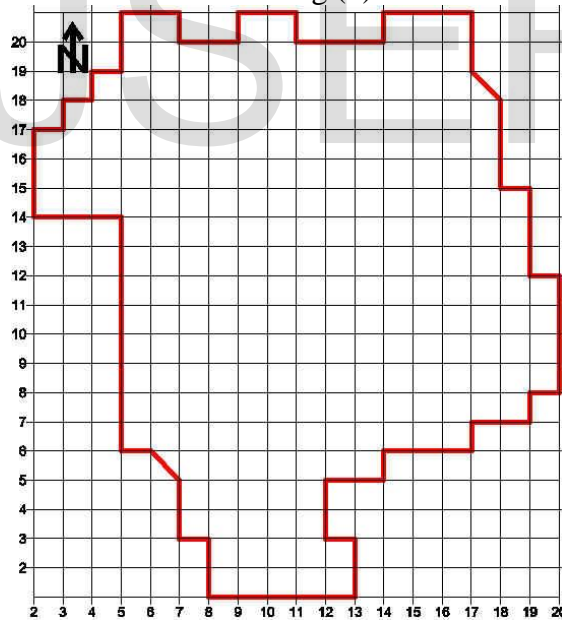


Fig.(7) Mesh Design of the Area

5.3 Input Data Files

Natural levels are obtained from arbitrary selected positions within the site domain. The measured levels locations are then matched over the discretized network of Fig.(7). After the natural levels have been specified for each mesh within the domain of Fig.(7), those are entered into the data input file (file no.1,(namly; *INPUT LEVELS.dat*) see the program, Appendix A). The matrix dimensions in x and y directions (NC & NR) in addition to the size of meshes are also chosen to be entered into the program (NC=20, NR=21 in the current case study, Appendix A). Mesh dimensions DX and DY is also entered into the program (DX=DY=250m in the current case study). The meshes without level readings, zero values are specified automatically for them by the program and basically not considered in the development of the required grading plane.

5.4 Output Data Files

Once, the model is run, the estimated optimum requested grading levels and cuts and fills depths are constituted in the output data files (3, 4, and 5, namely; *HNEW WORKSHEET*, *HNEW MATRIX*, and *CUTFILL* respectively, see the program, Appendix A).

6 Representation and Discussion of the Results

After the model has been run, new optimum levels is obtained from the output data file "*HNEW WORKSHEET.dat*" Table (1) and represented graphically in the contour map of Fig.(8). The figure shows regularity of the graded plane with a vertical interval equals =1m

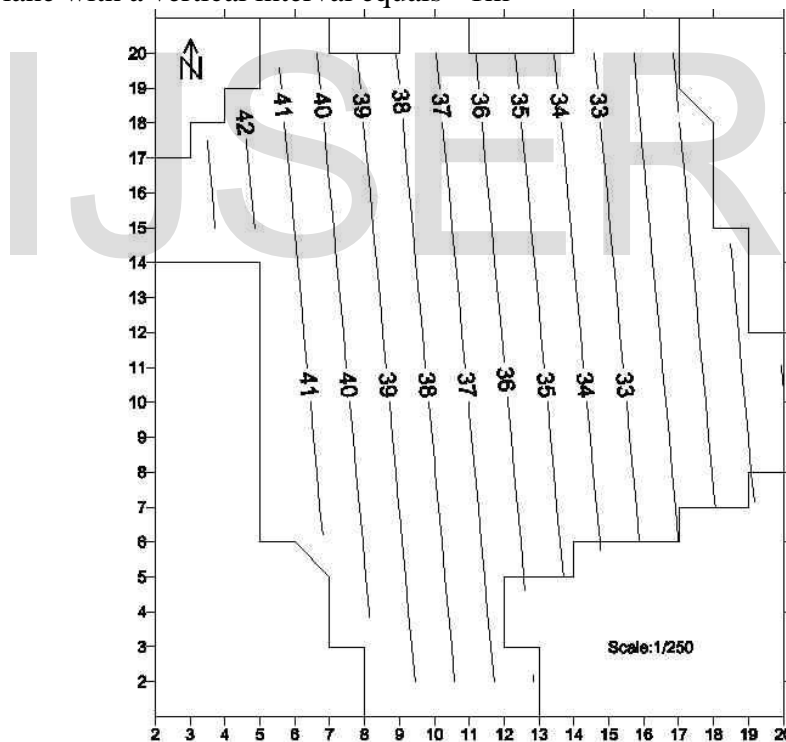


Fig.(8) Graded plane Contour Map of 1m Contour interval

Fig.(9) indicates the optimum graded optimum plane sketched in 3D. The model instructs that for this wide area ($17,562,500 \text{ m}^2$) the (error term) total summation of cut and fill earth work is 39.27 m^3 . This volume of cut and fill is relatively so small by comparing it with modeled area size.

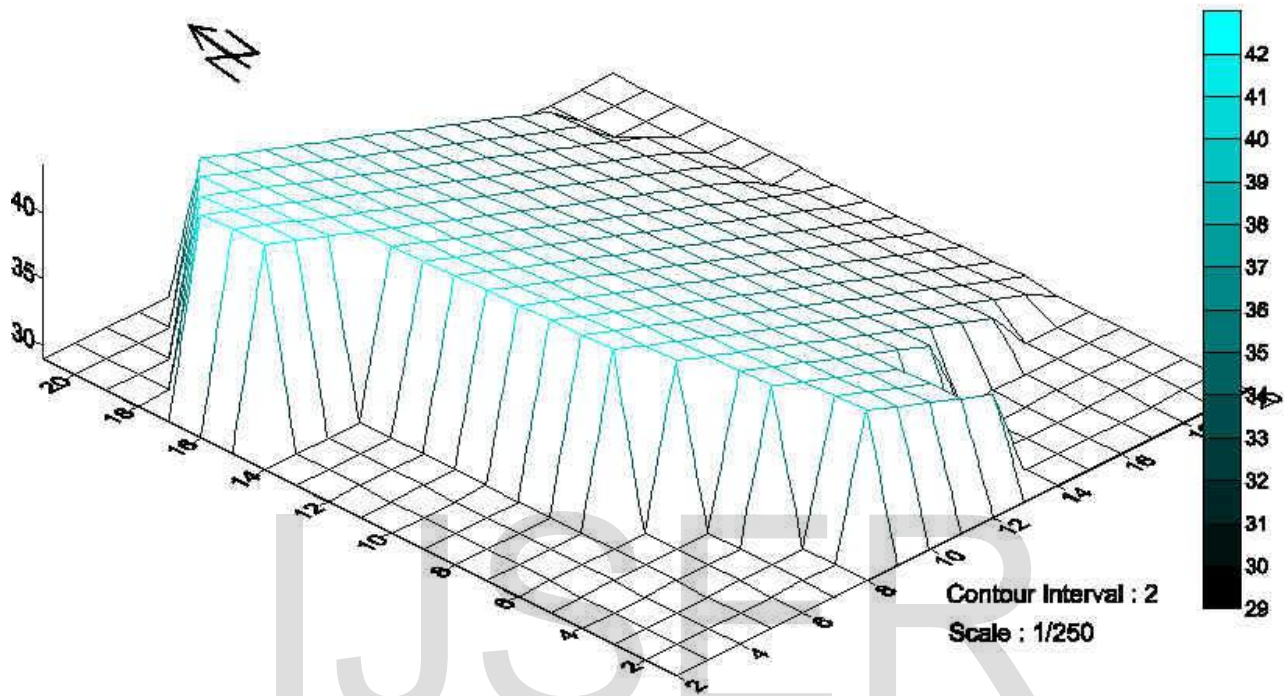


Fig.(9) 3D Graded Plane and Requested Surface

Table (1) Cut & Fill Depths Matrix in m (Minus sign means cut)

x	x	x	x	-2.3	3.38	4.5	x	4.08	3.44	2.9	x	x	2.33	1.55	0.65	-	0.21	x	x	x
x	x	x	x	-4.86	3.76	3.01	3.89	3.7	0	2.24	2.83	x	0	0	0	0.35	x	x	x	
x	x	x	-	-	-	-	1.89	3.17	3.21	2.32	3.44	2.62	1.78	0.87	0.05	0.62	-	-	x	x
x	x	7.22	14.11	17.29	15.88	3.58	1.75	3.24	3.31	2.94	2.7	1.94	0.76	0.02	-0.6	1.02	1.41	x	x	
x	1.23	1.78	-7.43	14.91	-6.9	3.12	3.85	3.98	3.62	3.03	2.33	1.35	0.17	0.86	1.42	1.44	1.55	x	x	
x	5.65	6.73	3.01	4.97	2.41	4.51	5.14	4.59	3.79	2.96	2.09	1.01	0.34	1.83	2.68	1.73	1.63	x	x	
x	7.84	8.29	7.86	7.72	6.78	6.25	5.67	4.68	3.61	2.69	1.92	1.01	0.36	2.25	4.76	2.63	1.53	x	x	
x	x	x	x	x	7.85	6.96	5.68	4.21	2.92	2.04	1.67	1.36	0.34	1.02	1.97	1.82	1.51	1.66	x	
x	x	x	x	x	7.64	6.89	5.03	3.06	1.58	0.76	0.91	2.05	1.14	0.41	0.4	0.58	1.05	1.46	x	
x	x	x	x	x	6.21	6.44	3.22	1.01	0.32	-1.1	0.99	1.13	1.11	0.84	0.54	0.05	0.67	1.23	x	
x	x	x	x	x	3.06	1.59	-x.54	1.99	2.43	2.22	1.38	0.11	0.56	0.68	0.49	0.07	-48	1.08	x	
x	x	x	x	x	-0.45	3.02	-5.26	5.45	4.46	3.19	1.86	0.65	0.16	0.48	0.42	0.1	-39	0.99	x	
x	x	x	x	x	-3.03	6.33	10.21	8.09	5.69	3.64	1.96	0.69	0.11	0.47	0.46	0.16	-32	0.92	x	
x	x	x	x	x	-4.08	6.79	-8.56	7.63	5.41	3.22	1.45	0.23	0.44	0.7	0.63	0.27	-24	0.84	x	
x	x	x	x	x	-3.88	5.81	-7.05	5.95	4.03	1.95	0.27	0.68	1.05	1.06	0.89	0.42	-16	x	x	
x	x	x	x	x	x	4.23	-4.67	3.88	2.21	0.19	1.45	1.88	1.72	1.4	1	0.48	x	x	x	
x	x	x	x	x	x	x	-2.69	-2	0.59	1.31	3.58	2.76	2.13	x	x	x	x	x	x	
x	x	x	x	x	x	x	-1.13	0.55	0.45	1.67	2.59	x	x	x	x	x	x	x	x	
x	x	x	x	x	x	x	x	0.47	1.06	1.5	1.71	x	x	x	x	x	x	x	x	
x	x	x	x	x	x	x	x	1.16	1.6	1.31	0.83	1.15	x	x	x	x	x	x	x	

Model Results, 2010

7 Conclusions

The following conclusions are obtained:-

- 1- The land grading mathematical model of the requested surface is an optimum and unique plane in which the summation of all cuts and fills within the modeled domain are accordingly minimized.
- 2- It is a simplified and applied practical and computational technology
- 3- The more measured levels in the input data files, the more accurate results of the requested surface.

8 Recommendations

The following points are recommended:-

- 1- A technical specialist who works in a wide areal surveying is recommended to use the new technology to save a labor and time.
- 2- Computer monitoring software may be developed to present the output data files rather than a Fortran Language programming

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Appendix-A

```
c
C
c
c
C RESULTS INSTRUCTION
c The sign of the results decides the cut or fill
c Negative means the value is cut
c Positive means the value is fill
PARAMETER (IMAX=40,jmax=40)
DIMENSION H(imax,jmax),HNEW(imax,jmax),Samir(IMAX,JMAX),
1CF(IMAX,JMAX)
```

```
OPEN(1,FILE='INPUT LEVELS.dat')
OPEN(2,FILE='SAMIR.dat')
OPEN(3,FILE='HNEW WORKSHEET.dat')
OPEN(4,FILE='HNEW MATRIX.dat')
OPEN(5,FILE='CUTFILL.dat')
```

DEFINITION OF FILES

- c *INPUT LEVELS* contains the measured Levels
- c *SAMIR* is a helpful file contains 1 & 0 value cells (has 1 for cells inside analyzed domain & 0 for cells outside domain)
- c *HNEW WORKSHEET* contains the output levels of the model in the form of Excel work sheet
- c *HNEW MATRIX* contains the output levels of the model in the form of matrix
- c *CUTFILL* contains the values of cut & fill

DATA INPUT

- c 1- The surveyer may take readings of a specified remarks with known spacing in the field.
 - c 2- Draw a map and design meshes
 - c 3- measure the distances between meshes in both x & y directions
 - c 4- calculate the number of grids in x direction and insert it to be NC
 - c 5- calculate the number of grids in y direction and insert it to be NR
 - c 6- estimate the distance between grids (in m) in both x and y direction insert them to be equal to Dx & Dy respectively
- ```
NC=20
NR=21
Dx=250
Dy=250
```

- c Where
- c NC: Number of columns, NR: Number of Rows
- c Dx & Dy :Grid spacing in meters

**READING OF DATA FILES**

```
DO I=1,NR
READ(1,20)(H(I,J),J=1,NC)
20 FORMAT(20F8.2)
ENDDO

DO 320 J=1,NR
320 READ(2,330)J,(SAMIR(I,J),I=1,NC)
330 FORMAT(I3,2X,20F5.1)
CONTINUE
```

**ESTIMATION OF AVERAGE LEVEL AND DOMAIN CENTROID**

```
22 SUMH=0.0
SUMHX=0.0
SUMHY=0.0
SUMX=0.0
SUMY=0.0
SUMXSE=0.0
SUMYSQ=0.0
DO 70 I=1,NR
DO 70 J=1,NC
IF(H(I,J).EQ.0.0) GOTO 70
KY=DY*I
KX=DX*J
N=N+1
SUMH=SUMH+H(I,J)
SUMHY=SUMHY+KY*H(I,J)
SUMHX=SUMHX+KX*H(I,J)
SUMX=SUMX+KX
SUMY=SUMY+KY
SUMXSQ=SUMXSQ+KX**2
SUMYSQ=SUMYSQ+KY**2
70 ENDDO
AVEH=SUMH/N
```

```
 CENX=SUMX/N
 CENY=SUMY/N
c AVEH IS THE AVERAGE LEVEL
C CENX IS THE CENTER OF THE GRID SYSTEM IN X DIRECTION
C CENY IS THE CENTER OF THE GRID SYSTEM IN Y DIRECTION

C CALCULATION OF SLOPES
 SX=(SUMHX- N*CENX*AVEH)/(SUMXSQ-N*(CENX)**2)
 SY=(SUMHY- N*CENY*AVEH)/(SUMYSQ-N*(CENY)**2)

C CALCULATIONS OF NEW LEVELS
 DO 80 I=1,NR
 DO 80 J=1,NC
C DIFX & DIFY ARE DISTANCES BETWEEN CENTROID & THE CONSIDERED GRID
 DIFX=J*DX-CENX
 DIFY=I*DY-CENY
 HNEW(I,J)=AVEH+SX*DIFX+SY*DIFY
80 ENDDO

 DO 110 I=1,NC
 DO 110 J=1,NR
 WRITE(3,*)I,(NR+1-J),HNEW(I,J)
195 FORMAT(20F4.2)
110 ENDDO
 DO 326 J=1,NR
326 WRITE(4,336)(HNEW(I,J),I=1,NC)
336 FORMAT(20F8.2)

C CALCULATIONS OF CUTS & FILLS
 DO 426 I=1,NC
 DO 426 J=1,NR
 CF(I,J)=HNEW(I,J)-H(I,J)
 IF(SAMIR(I,J).EQ.0.0)CF(I,J)=0.0
426 Continue
 DO 526 J=1,NR
526 WRITE(5,536)(CF(I,J),I=1,NC)
536 FORMAT(20F8.2)

c SUMMATION OF CUT & FILL
 DO 190 I=1,NR
 DO 190 J=1,NC
 Sk=H(I,J)
 IF(SK.EQ.0.0)GOTO 190
 SUM=SUM+CF(I,J)
190 ENDDO
 WRITE(*,*)SUM
200 END
```

